MATHEMATICS

> There is no branch of mathematics, however abstract, which may not some day be applied to phenomena of the real world.
>
> Nikolai Lobatchevsky (1792 - 1856)

A knot may seem a simple, everyday thing - at least to the non-mathematical mind. We encounter knots when we tie our shoe laces, when we try to untangle our Christmas tree lights or even when we comb our hair (provided it is long enough). So what's so interesting about knots?

First of all, these examples do not really describe what a mathematician would call a knot: in the mathematical discipline of topology, a knot is defined as a *closed*, non-self-intersecting curve embedded in three dimensional space. A "knot" that has loose ends is - from a topologist's point of view - trivial and thus rather uninteresting, even if the examples given above led you to assume the opposite. An assembly of knots with mutual entanglements is called a link.

Although knots have been around for thousands of years, the interest in them received a major impetus in the late 19th century due to expected connections with chemistry. At that time, many scientists believed that the universe was filled with a mysterious substance called ether. William Thomson (later Lord Kelvin) introduced the idea that each chemical element was entangled with the ether in a characteristic way, therefore making it possible to describe the structure of matter with the aid of knot theory. Soon thereafter, P. G. Tait started to catalogue possible knots by trial and error. However, as a consequence of the atomic revolution the theory of ether was soon dismissed, leaving the mathematicians alone with their knots. This however did not really diminish their interest in the subject.

Much research has been done in the meantime, trying to find mathematical ways to describe different knots. This is usually done by defining so-called "knot invariants". These are functions from the set of all knots to some other set such that their value does not change if we transform the knot into another "equivalent" knot. Thus, if such a function takes different values on two knots we know that they are not equivalent, but there may well be non-equivalent knots on which the function takes the same value. Examples for such invariants are the Vassiliev invariants, the Alexander polynomial or the torsion number.

Knot theory has found applications in a variety of areas that seem to be quite unrelated at first sight. One example is the study of DNA: the double helix is so long that it can be tangled up millions of times inside the cell. This fact has an interesting impact on the properties of the resulting strand after replication. Knot theory also provides some interesting puzzles for statistical mechanics. This application was left undiscovered until the 1980's, when Vaughan Jones realized the connection while discovering a new polynomial invariant for knots, now known as the Jones polynomial. Other examples of applications can be found in molecular chemistry and particle physics.

The article by Pedro Manchón that is included in this section of the *Annals* studies a special type of link, the so-called pretzel links, and gives a closed formula for their Kauffman bracket. As the author explains, the Kauffman bracket is closely related to the Jones polynomial that we mentioned above.

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