

PARADOX OF REDISTRIBUTION IN POLISH POLITICS

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Abstract

In this study, all the parliamentary elections held in Poland since 1989 were examined. The occurrence of the redistribution paradox for five kinds of power indices was investigated for Polish elections in 1989, 1991, 1993, and 1997.

The power indices analyzed were the Shapley-Shubik index, the normalized and absolute Banzhaf indices, the Deegan-Packel index, and the Holler-Packel index.

Keywords: power index, paradox of redistribution

1. Introduction

In Game Theory, a voting situation can be modeled as a *voting game*, that is, as a pair (N, v) , where $N = \{1, \dots, n\}$ denotes the set of players and v is a certain function assigning either 0 or 1 to any subset of N . Moreover, $v(N)=1$ and v is non-decreasing, i.e., $v(S) \leq v(T)$ whenever $S \subseteq T \subset N$. Any nonempty subset of N is called a *coalition*. A coalition S is

winning if $v(S) = 1$, and *losing* if $v(S) = 0$. A *minimal winning coalition* is a winning coalition such that no proper subset of it is winning. One interesting class of voting games is the class of *weighted voting games*, denoted by $[q; w_1, w_2, \dots, w_n]$, where q is the *quota* needed for a coalition to win, and w_i is the number of

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votes of player i ($i=1,2,\dots,n$). Then, we have $v(S) = 1$ if $\sum_{i \in S} w_i \geq q$, and $v(S) = 0$ otherwise.

The power of players can be measured by *power indices*. A power index is a certain vector $\phi = (\phi_1, \dots, \phi_n)$, where ϕ_i is interpreted as a measure of the influence that player i ($i=1,2,\dots,n$) can exert on the outcome. Many ideas about how to evaluate the distribution of power among the players have appeared. For an exposition of the intuitive meaning of the different power indices, see [7]. In this study, the following power indices were analyzed:

- *Shapley-Shubik index* (Shapley and Shubik, 1954)

$$Sh_i(v) = \sum_{\{S \subseteq N: i \in S\}} \frac{(s-1)!(n-s)!}{n!} [v(S) - v(S - \{i\})]$$

for each $i = 1, \dots, n$, where s denotes the number of players in coalition S

- *normalized Banzhaf index* (Banzhaf, 1965) - for each $i = 1, \dots, n$

$$Bz_i(v) = \frac{\sum_{\{S \subseteq N: i \in S\}} [v(S) - v(S - \{i\})]}{\sum_{k \in N} \sum_{\{S \subseteq N: i \in S\}} [v(S) - v(S - \{i\})]}$$

- *absolute Banzhaf index (non-normalized Banzhaf index, Dubey and Shapley, 1979)* - for each $i = 1, \dots, n$

$$nnBz_i(v) = \frac{1}{2^{n-1}} \sum_{\{S \subseteq N: i \in S\}} [v(S) - v(S - \{i\})]$$

- *Deegan-Packel index* (Deegan and Packel, 1978)

$$DP_i(v) = \frac{1}{m} \sum_{\{S \in M: i \in S\}} \frac{1}{|S|} [v(S) - v(S - \{i\})]$$

for each $i = 1, \dots, n$, where M and m denote the set of all minimal winning coalitions and the total number of minimal winning coalitions, respectively

- *Holler-Packel index* (Holler and Packel, 1983)

$$HP_i(v) = \frac{nnHP_i(v)}{\sum_{k \in N} nnHP_k(v)}$$

for $i = 1, \dots, n$, where

$$nnHP_i(v) = \frac{1}{m} \sum_{\{S \in M: i \in S\}} [v(S) - v(S - \{i\})]$$

In the study and calculation of power indices, some unexpected and counterintuitive features emerged, called *paradoxes of voting power*. One of the well-known paradoxes is the *paradox of redistribution*, which was introduced by Fischer and Schotter (1978). A power index displays the redistribution paradox when either a party's voting weight decreases and at the same time its power index increases, or when a party gains in terms of voting weight, but loses in voting power. I will denote these paradoxical situations as (P1) and (P2), respectively. Formally, we can describe them as follows.

We have two weighted voting games $V = [q; w_1, w_2, \dots, w_n]$ and

$V' = [q; w'_1, w'_2, \dots, w'_n]$, where

$$\sum_{i=1}^n w_i = \sum_{i=1}^n w'_i. \text{ A power index } \phi \text{ displays the}$$

redistribution paradox:

(P1) if for some i , $w'_i < w_i$ and $\phi_i(V') > \phi_i(V)$ or

(P2) if for some j , $w'_j > w_j$ and $\phi_j(V') < \phi_j(V)$.

Power indices have been applied to political institutions or elections. In particular, Sosnowska (1996, 1999) applied the modified Shapley-Shubik index (the Shapley-Shubik index for games with a priori unions) to elections in Poland and defined the generalized paradox of redistribution concerning the modified Shapley-Shubik index. In this study, parliamentary elections held in Poland in 1989, 1991, 1993, and 1997 were examined. The occurrence of the redistribution paradox for five power indices was investigated, that is, for the Shapley-Shubik index, the normalized and non-normalized Banzhaf indices, the Deegan-Packel index, and the Holler-Packel index.

2. Parliamentary Elections in Poland

The year 1989 was crucial for Poland, starting the transition from People's Poland to democratic Poland. Table 1 (in Appendix) presents an example of the distribution of power in the Sejm of the Polish People's Republic before 1989. At that time the main role on the political scene was played by the communist party, the *Polish United Workers' Party (PZPR)*. Its allies were the *United Peasant Party (ZSL)* and the *Democratic Party (SD)*. From Table 1, we can conclude that the PZPR, having 55.5% of the parliamentary seats, was itself a winning

coalition. The 1989 parliamentary elections were the beginning of free elections in Poland. "The Round Table Talks" - debates between communists and Solidarity (the anti-communist opposition) resulted in an agreement concerning fundamental changes in the political system in Poland. According to this agreement, 60% of the seats in the Lower House of Parliament (Sejm) was assigned to the ruling coalition (PZPR, ZSL, SD), 5% for the others - PAX, PZKS, UChS (three small religious organizations controlled by the communist party), and 35% of the seats was assigned to free elections. A new parliament was elected in June 1989. The anti-communist opposition - the *Citizens' Parliamentary Club (OKP)* received all 35% of the available seats. Table 2 (see Appendix) presents the distribution of seats and the power indices of the parties in the Lower House of Polish Parliament after the 1989 elections. At the end of 1989, the ZSL started to change its alliances. In particular, it changed its name.

The Polish Peasant Party (PSL) was the successor of the ZSL. Many non-communist groups with a history of underground activities could now be openly active as political parties. At the end of 1990, the *Democratic Union (UD)*, a centrist party with a liberal economic program, was formed. In 1991, left-wing organizations and groups formed the *Alliance of the Democratic Left (SLD)*. Its main goal was maximizing the socialist vote in the forthcoming parliamentary elections to be held in October 1991. In general terms, these elections marked the end of the transition from People's Poland to democratic Poland. Formally, a victory belonged to the UD, the party of the intelligentsia, but it was a limited

success. The SLD ended as second, only slightly below the UD. The PSL consolidated its position as the main party of the rural electorate. Table 3 shows the distribution of seats and the power indices of the parties in the Lower House after the 1991 elections. Again, a new parliament was elected in September 1993. The elections were a victory for the non-Solidarity parties. In Poland, as in most of the countries of Central and Eastern Europe, the difficulties of the economic transition produced a swing to the left. Table 4 presents the distribution of seats and the power indices of the parties in the Lower House of Polish Parliament after the 1993 elections. After the 1993 elections, the SLD together with the PSL had an absolute majority in the Sejm. In 1994-95, there were several initiatives designed to bring the right-wing parties together. In late 1995, the "*Election Action "Solidarity" (AWS)*" was formed. The left-wing coalition was becoming increasingly unpopular. The next parliamentary elections in Poland were held in September 1997. These elections were a victory for the AWS. Table 5 shows the distribution of seats and the power indices of the parties in the Lower House after the 1997 elections.

The main aim of this study was to check the redistribution paradox in Polish politics. The analysis of Tables 2-5 leads to the conclusion that the paradox mentioned appears quite frequently for Polish parties. Table 6 (see Appendix) shows this paradox for the Lower House of Polish Parliament. The Shapley-Shubik index, the normalized and non-normalized Banzhaf indices, the Holler-Packel index, and the Deegan-Packel index were analyzed. The redistribution paradox appeared for the

ZSL/PSL, the SD, the SLD, the UD, and the *German Minority (MN)*. A comparison of the data for ZSL/PSL in 1989 and 1993 showed that paradox (P2) was found for all five power indices mentioned. For the PSL in 1991 and 1997, paradox (P1) appears for the Holler-Packel and the Deegan-Packel indices. All power indices of the SD display (P1) for the elections in 1989 and 1991. For the SLD, the redistribution paradox has been found twice: (P2) in 1991/1997 for the absolute Banzhaf index and (P1) in 1993/1997 for the Holler-Packel index. Paradox (P2) appears for the absolute Banzhaf index of the UD, when comparing the elections in 1991 and 1993. Finally, for the MN, paradox (P1) has been found three times: for the Shapley-Shubik and normalized Banzhaf indices in 1991/1993, 1991/1997, 1993/1997 and, moreover, for the Holler-Packel and Deegan-Packel indices in 1991/1993 and 1991/1997.

3. References

- [1] J. Banzhaf, Weighted Voting Doesn't Work: A Mathematical Analysis, *Rutgers Law Review* 19 (1965) 317-343
- [2] S. Berglund, T. Hellen and F. Aarebrot (eds), *The Handbook of Political Change in Eastern Europe*, Edward Elgar, 1998
- [3] J. Deegan and E.W. Packel, A New Index of Power for Simple n-Person Games, *International Journal of Game Theory* 7 (1978) 113-123
- [4] P. Dubey and L.S. Shapley, Mathematical Properties of the Banzhaf Power Index, *Mathematics of Operations Research* 4 (1979) 99-131

- [5] D. Fischer and A. Schotter, The Inevitability of the Paradox of Redistribution in the Allocation of Voting Weights, *Public Choice* 33 (1978) 49-67
- [6] M.J. Holler and E.W. Packel, Power, Luck and the Right Index, *Journal of Economics* 43 (1983) 21-29
- [7] A. Laruelle, On the Choice of a Power Index, Universidad de Alicante, Alicante, Spain (2000)
- [8] L.S. Shapley, A Value for n-Person Games, *Annals of Mathematics Studies* 28 (1953) 307-317
- [9] L.S. Shapley and M. Shubik, A Method for Evaluating the Distribution of Power in a Committee System, *American Political Science Review* 48 (1954) 787-792
- [10] H. Sosnowska, Shapley Values of Games with a Priori Unions as a Method of Analysis of Elections in Poland 1989-1994, Warsaw School of Economics, discussion paper, Warsaw, Poland (1996)
- [11] H. Sosnowska, Generalized Paradox of Redistribution and Size. Application to Polish Elections in 1997, Warsaw School of Economics, discussion paper, Warsaw, Poland (1999)

MN - German Minority
 OKP - Citizens' Parliamentary Club
 PSL - Polish Peasant Party
 PZPR - Polish United Workers' Party
 ROP - Movement for Rebuilding Poland
 RDS - Social Democratic Movement
 SD - Democratic Party
 SL - Peasant Alliance
 SLD - Alliance of the Democratic Left
 UD - Democratic Union
 UP - Union of Labour
 UPR - Union of Real Politics
 UW - Union of Freedom
 ZSL - United Peasant Party

4. Appendix

The names of the parties appearing in Tables 1 - 6 are the following:

AWS - Election Action "Solidarity"

BBWR - Non-Partisan Bloc for Supporting Reforms

KLD - Liberal-Democratic Congress

KPN - Confederation for an Independent Poland

Table 1 - the Sejm of the Polish People's Republic in its VI term

Parliamentary club	PZPR	ZSL	SD	Znak	PAX	ChSS	other
% of seats	55.5	25.5	8.6	1	1	0.4	8

Let:

Sh - the Shapley-Shubik index of a party

Bz - the normalized Banzhaf index of a party

nnBz - the non-normalized Banzhaf index of a party

HP - the Holler-Packel index of a party

DP - the Deegan-Packel index of a party

Table 2 - Parliamentary elections in Poland in 1989 (the Lower House, 460 seats)

Party	Weight	% of seats	<i>Sh</i>	<i>Bz</i>	<i>nnBz</i>	<i>HP</i>	<i>DP</i>
PZPR	173	37.6	0.3333	0.3333	0.5	0.3333	0.3333
OKP	161	35.0	0.3333	0.3333	0.5	0.3333	0.3333
ZSL	76	16.5	0.3333	0.3333	0.5	0.3333	0.3333
SD	27	5.9	0	0	0	0	0
Others	23	5.0	0	0	0	0	0

Others - PAX, PZKS, UChS

Table 3 - Parliamentary elections in Poland in 1991 (the Lower House, 460 seats)

Party	Weight	<i>Sh</i>	<i>Bz</i>	<i>nnBz</i>	<i>HP</i>	<i>DP</i>
UD	62	0.1402	0.139	0.3596	0.0422	0.0445
SLD	60	0.1351	0.1341	0.3467	0.0423	0.0446
KPN	51	0.1122	0.1113	0.2877	0.0418	0.0432
PSL	50	0.1097	0.1089	0.2817	0.0419	0.0432
Cath Elec. Action	50	0.1097	0.1089	0.2817	0.0419	0.0432
Alliance Centrum	44	0.0946	0,0948	0.2452	0.0419	0.0431
KLD	37	0.0795	0.0802	0.2074	0.0423	0.0433
SL	28	0.0594	0.0596	0.154	0.042	0.0422
Solidarity	27	0.0577	0.0581	0.1502	0.0421	0.0425
P. Friends of Bier	16	0.0321	0.0329	0.0851	0.0417	0.0416
MN	7	0.014	0.0143	0.0371	0.0414	0.0408
Christian Democra.	5	0.01	0.0103	0.0267	0.0417	0.041
Solidarity of Labour	4	0.008	0.0083	0.0214	0.0417	0.041
Pol. Chris. Democra.	4	0.008	0.0083	0.0214	0.0417	0.041
UPR	3	0.006	0.0062	0.0161	0.0417	0.041
Party X	3	0.006	0.0062	0.0161	0.0417	0.041
Mov. of Siles. Aut.	2	0.004	0.0041	0.0107	0.0415	0.0407
SD	1	0.002	0.0021	0.0054	0.0412	0.0403
RDS	1	0.002	0.0021	0.0054	0.0412	0.0403
Mount. Leaque	1	0.002	0.0021	0.0054	0.0412	0.0403
Great-Pol. Soc. Un.	1	0.002	0.0021	0.0054	0.0412	0.0403

Social Chris. Un.	1	0.002	0.0021	0.0054	0.0412	0.0403
Solidarity “80”	1	0.002	0.0021	0.0054	0.0412	0.0403
Union of Great Pol.	1	0.002	0.0021	0.0054	0.0412	0.0403

Table 4 - Parliamentary elections in Poland in 1993 (the Lower House, 460 seats)

Party	weight	<i>Sh</i>	<i>Bz</i>	<i>nnBz</i>	<i>HP</i>	<i>DP</i>
SLD	171	0.4143	0.3962	0.6563	0.1818	0.2262
PSL	132	0.2143	0.2075	0.3438	0.1818	0.1905
UD	74	0.2143	0.2075	0.3438	0.1818	0.1905
UP	41	0.081	0.0943	0.1563	0.1364	0.131
KPN	22	0.0476	0.0566	0.0938	0.1364	0.119
BBWR	16	0.0143	0.0189	0.0313	0.0909	0.0714
MN	4	0.0143	0.0189	0.0313	0.0909	0.0714

Table 5 - Parliamentary elections in Poland in 1997 (the Lower House, 460 seats)

Party	weight	<i>Sh</i>	<i>Bz</i>	<i>nnBz</i>	<i>HP</i>	<i>DP</i>
AWS	201	0.4167	0.4038	0.6563	0.2143	0.2667
SLD	164	0.2167	0.2115	0.3438	0.2143	0.2167
UW	60	0.2167	0.2115	0.3438	0.2143	0.2167
PSL	27	0.0833	0.0962	0.1563	0.1429	0.1333
ROP	6	0.05	0.0577	0.0938	0.1429	0.1167
MN	2	0.0167	0.0192	0.0313	0.0714	0.05

Table 6 - Paradox of redistribution for Polish parties (the Lower House)

party	ZSL/ PSL	PSL	SD	SLD	SLD	UD	MN	MN	MN
x	1989	1991	1989	1991	1993	1991	1991	1991	1993
y	1993	1997	1991	1997	1997	1993	1993	1997	1997
w_x	76	50	27	60	171	62	7	7	4
w_y	132	27	1	164	164	74	4	2	2
Sh_x	0.3333		0				0.014	0.014	0.0143
Sh_y	0.2143		0.002				0.0143	0.0167	0.0167
Bz_x	0.3333		0				0.0143	0.0143	0.0189
Bz_y	0.2075		0.0021				0.0189	0.0192	0.0192
$nnBz_x$	0.5		0	0.3467		0.3596			
$nnBz_y$	0.3438		0.0054	0.3438		0.3438			
HP_x	0.3333	0.0419	0		0.1818		0.0414	0.0414	
HP_y	0.1818	0.1429	0.0412		0.2143		0.0909	0.0714	
DP_x	0.3333	0.0432	0				0.0408	0.0408	
DP_y	0.1905	0.1333	0.0403				0.0714	0.05	